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Short note

Spin-orbit-like terms in semileptonic weak Hamiltonian

A.L. Barabanov

Kurchatov Institute, 123182 Moscow, Russia

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Abstract. It is shown that new spin-orbit-like terms appear in the effective nonrelativistic weak Hamiltonian for nucleon provided that nuclear potential is taken into account. Arguments for their considerable enhancement, in particular, in relativistic nuclear model of Walecka are advanced.

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Effective weak Hamiltonian for nucleon is given by covariant product of lepton current J_λ and nucleon current operator Γ_λ. To use nonrelativistic nucleon wave functions one puts effective Hamiltonian to nonrelativistic form by Foldy–Wouthuysen (FW) transformation (see, e.g., [1]). Nonrelativistic Hamiltonian is a power series in k_λ/M, where k_λ is the 4-vector of transferred momentum, and M is the nucleon mass. To describe β-decay, where an energy release is about $10^{-3}M$, one uses only zero order terms. Whereas first order terms are of importance for muon capture, where an energy release is about $10^{-1}M$, as well as for neutrino-induced reactions involving transferred momenta of the same order of magnitude.

For the sake of definiteness we shall consider muon capture. Before capture a muon is in the 1s state of mesic atom and is described by the 4-component wave function $\psi_\mu(\mathbf{r}_\mu, t)$, E_μ is the total energy of muon. A final neutrino with momentum $\mathbf{k}_\nu$ and energy $E_\nu = k_\nu$ is described by the 4-component wave function $\psi_\nu(\mathbf{r}_\nu, t)$. Assuming that the weak nucleon-lepton interaction is pointlike one obtains for the lepton current

$$J_\lambda = i\psi_\nu^+(\mathbf{r}, t)\gamma_4\gamma_\lambda(1+\gamma_5)\psi_\mu(\mathbf{r}, t). \quad (1)$$

Then the effective relativistic Hamiltonian can be written in the form

$$H_W = \frac{G\cos\theta_C}{\sqrt{2}} iJ_\lambda \Gamma_\lambda \tau_-, \quad (2)$$

where G is the weak-interaction coupling constant, θ_C is the Cabibbo angle, and the lowering operator τ_- transforms a proton into a neutron. The operator of the weak nucleon current is given by (see, e.g., [2])

$$\Gamma_\lambda = \gamma_4\left(g_V\gamma_\lambda + \frac{g_M}{2M}\sigma_{\lambda\rho}k_\rho - g_A\gamma_\lambda\gamma_5 - i\frac{g_P}{m}k_\lambda\gamma_5\right). \quad (3)$$

Here $\sigma_{\lambda\rho} = (\gamma_\lambda\gamma_\rho - \gamma_\rho\gamma_\lambda)/2i$, m is the muon mass, and $k_\lambda = (\mathbf{k}_\nu, -i(E_\mu - E_\nu))$ is the 4-momentum transfer. The form factors of vector interaction g_V, axial-vector interaction g_A, weak magnetism g_M, and induced pseudoscalar interaction g_P depend on $k^2 = k_\lambda k_\lambda$. We omit the contribution of the second class currents (i.e. scalar and tensor terms).

One usually performs FW transformation for free nucleon describing by Hamiltonian $H_N = M\beta + \boldsymbol{\alpha}\mathbf{p}$. First order terms in $1/M$ were first obtained in [3], whereas the second order corrections $\sim 1/M^2$ were calculated in [4,5] (see also [6,7]). However, the nucleons inside a nucleus are not, in fact, free.

Let us take the one-nucleon relativistic Hamiltonian in the form

$$H = M\beta + \boldsymbol{\alpha}\mathbf{p} + U(r) + H_W, \quad (4)$$

where nucleon-nucleus potential $U(r)$ is assumed for simplicity to be of a central type. According to FW procedure, $H - M\beta$ has to be presented as the sum of even $\mathcal{E}$ and odd $\mathcal{O}$ parts. Then the nonrelativistic Hamiltonian takes the form

$$H' = M + \mathcal{E} + \frac{\beta\mathcal{O}^2}{2M} - \frac{[\mathcal{O}, [\mathcal{O}, \mathcal{E}]]}{8M^2} - \frac{i[\mathcal{O}, \dot{\mathcal{O}}]}{8M^2} + \ldots, \quad (5)$$

where $[A, B] = AB - BA$. It is well known that crossing of the operator $\boldsymbol{\alpha}\mathbf{p}$ entering $\mathcal{O}$ with the potential $U(r)$, which belongs to $\mathcal{E}$, in the term $\sim [\mathcal{O}, [\mathcal{O}, \mathcal{E}]]$ leads to Darwin and spin-orbit interactions being of the second order in $1/M$. In a similar manner crossing of odd operators from H_W with $U(r)$ gives additional second order terms, which never took into account.

The result of FW transformation of the Hamiltonian (4) is of the form

$$H' = M + \frac{\mathbf{p}^2}{2M} + U(r) + \frac{\Delta U(r)}{8M^2} + \frac{U'(r)}{4M^2 r}(\boldsymbol{\sigma}[\mathbf{r}\times\mathbf{p}]) + H'_W, \quad (6)$$

$$H'_W = \frac{G\cos\theta_C}{\sqrt{2}}\Big(iJ_4\Big[G_V + G_P(\boldsymbol{\sigma}\mathbf{n}_\nu) + \\ + g_A(\boldsymbol{\sigma}\frac{\mathbf{p}}{M}) + g_P\frac{iU'(r)}{4M^2 r}(\boldsymbol{\sigma}\mathbf{r})\Big] + \\ + \mathbf{J}\Big[G_A\boldsymbol{\sigma} + g_V\frac{\mathbf{p}}{M} + g_V\frac{U'(r)}{4M^2 r}[\boldsymbol{\sigma}\times\mathbf{r}]\Big]\Big)\tau_-, \tag{7}$$

where $\mathbf{n}_\nu = \mathbf{k}_\nu/k_\nu$. Nonrelativistic weak Hamiltonian (7) includes the known zero and first order terms [2] and two new spin-orbit-like second order terms, which are proportional to $U'(r)$. We omit all other second order terms obtained in [4,5]. Usual notations for renormalized form factors are used

$$G_V = g_V(1+\frac{E_\nu}{2M}), \quad G_P = \frac{E_\nu}{2M}(g_P - g_A - g_V - g_M), \\ G_A = g_A - \frac{E_\nu}{2M}(g_V + g_M). \tag{8}$$

It is worth noting now that nuclear spin-orbit coupling is enhanced as compared to the term entering (6) by the factor of ~ 20 (and has the opposite sign) [8]. Thus, one may hope for a similar enhancement of the spin-orbit-like terms in the weak Hamiltonian (7). It is of importance because the second order terms enhanced by the factor of ~ 20 would be of the same order of magnitude than the first order terms.

To demonstrate the feasibility for such effect one can address to relativistic nuclear model of Walecka [9,10]. In its simplest version a nucleon interacts with meson mean fields, one of which, $\Phi(r)$, is scalar, and the other, $V(r)$, is timelike component of 4-vector. The one-nucleon relativistic Hamiltonian takes the form

$$H = M\beta + \boldsymbol{\alpha}\mathbf{p} + V(r) - \Phi(r)\beta + H_W. \tag{9}$$

Both functions $V(r)$ and $\Phi(r)$ are positive, however, as it is seen from (9), $\Phi(r)$ and $V(r)$ represent attractive and repulsive potentials, respectively. Both potentials are very strong, e.g., $V(0) \simeq 0.37M$ and $\Phi(0) \simeq 0.45M$ for ^{40}Ca nucleus [11], but they almost cancel in (9).

This model describes the magnitude and the sign of nuclear spin-orbit coupling [12]. Indeed, FW transformation of (9) gives

$$H' = M + \frac{\mathbf{p}^2}{2M} + V(r) - \Phi(r) + \frac{\Delta V(r)}{8M^2} - \\ - \frac{\{\boldsymbol{\nabla},\{\boldsymbol{\nabla},\Phi(r)\}\}}{8M^2} + \frac{V'(r)+\Phi'(r)}{4M^2 r}(\boldsymbol{\sigma}[\mathbf{r}\times\mathbf{p}]) + H'_W, \tag{10}$$

where $\{A,B\} = AB+BA$. It is seen that scalar and vector contributions add up in the spin-orbit term, resulting in its enhancement.

So, the question is, does the same summation arises for the spin-orbit-like terms in H'_W? It turns out that this is the case. The weak Hamiltonian is of the form (7) with $U'(r) \to V'(r) + \Phi'(r)$ if one neglects corrections $\sim \Phi/M$ to the form factors.

Thus, it is shown that new spin-orbit-like terms appear in the effective nonrelativistic weak Hamiltonian for nucleon provided that nuclear potential is taken into account. Being enhanced by the factor of ~ 20, they may give the contribution of the same order of magnitude than the first order terms, usually allowed for. Finally, it is shown that such enhancement really arises in the Walecka model.

New terms in weak Hamiltonian may result in an improvement of description of some muon capture data. For instance, the results obtained in [13] for ^{28}Si nucleus point out an anomalous low value of g_P. On the other hand, search for new terms contribution to the weak semileptonic interaction may be considered as a test for relativistic nuclear model. The other test based on a lowering of the threshold for $p\bar{p}$ production on a nucleus was recently proposed in [14].

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